THE APRIL MEETING OF THE AMERICAN MATHEMATICAL SOCIETY IN NEW YORK.

THE two hundred and tenth regular meeting of the Society was held at Columbia University on Saturday, April 24, 1920, extending through the usual morning and afternoon sessions. The total attendance numbered over one hundred and thirty, and included the following eighty-two members:

Dr. J. W. Alexander, Professor R. C. Archibald, Dr. I. A. Barnett, Professor F. W. Beal, Mr. D. R. Belcher, Professor Suzan R. Benedict, Professor A. A. Bennett, Professor W. J. Berry, Mr. William Betz, Professor Joseph Bowden, Professor R. W. Burgess, Professor B. H. Camp, Professor C. W. Cobb, Professor A. Cohen, Professor F. N. Cole, Professor Elizabeth B. Cowley, Professor E. S. Crawley, Dr. W. L. Crum, Professor Louise D. Cummings, Professor L. E. Dickson, Professor L. P. Eisenhart, Professor H. B. Fine, Professor T. S. Fiske, Professor W. B. Fite, Dr. L. R. Ford, Professor O. E. Glenn, Dr. T. H. Gronwall, Professor Olive C. Hazlett, Professor E. R. Hedrick, Dr. A. A. Himwich, Professor T. R. Hollcroft, Professor E. V. Huntington, Mr. S. A. Joffe, Professor Edward Kasner, Professor O. D. Kellogg, Professor C. J. Keyser, Dr. E. A. T. Kircher, Dr. K. W. Lamson, Mr. Harry Langman, Professor P. H. Linehan, Mr. L. L. Locke, Professor W. R. Longley, Mr. John McDonnell, Professor C. R. MacInnes, Professor H. P. Manning, Dr. A. L. Miller, Professor H. H. Mitchell, Professor R. L. Moore, Professor Frank Morley, Professor G. W. Mullins, Professor W. F. Osgood, Mr. George Paaswell, Dr. Alexander Pell, Professor Anna J. Pell, Dr. G. A. Pfeiffer, Professor Susan M. Rambo, Professor H. W. Reddick, Professor L. W. Reid, Mr. L. H. Rice, Professor R. G. D. Richardson, Dr. J. F. Ritt, Professor J. E. Rowe, Professor Paul Saurel, Dr. Caroline E. Seely, Professor S. P. Shugert, Professor P. F. Smith, Professor C. M. Sparrow, Dr. J. M. Stetson, Mr. J. J. Tanzola, Professor J. I. Tracey, Mr. H. S. Vandiver, Professor E. B. Van Vleck, Professor Oswald Veblen, Mr. A. L. Wechsler, Mr. R. A. Wetzel, Professor H. S. White, Professor J. K. Whittemore, Miss Ella C. Williams, Professor F. B. Williams, Professor A. H. Wilson, Professor Ruth G. Wood, President R. S. Woodward.

President Frank Morley occupied the chair, yielding it to Ex-President R. S. Woodward during the presentation of the papers on relativity at the afternoon session. The Council announced the election of the following persons to membership in the Society: Professor H. S. Everett, Bucknell University; Dr. L. J. Rouse, University of Michigan; Professor Nilos Sakellariou, University of Athens; Mr. H. L. Smith, University of Wisconsin; Professor Eugene Taylor, University of Wisconsin; Professor W. P. Webber, University of Pittsburgh. Thirteen applications for membership in the Society were received.

Professor L. P. Eisenhart was reelected a member of the Editorial Committee of the *Transactions* for a term of three years beginning October 1, 1920. Professor P. F. Smith will retire from the Editorial Committee on October 1, after nine years' service as editor, and Professor G. D. Birkhoff will fill out Professor Smith's unexpired term. Professor Oswald Veblen was appointed representative of the Society in the Division of Physics of the National Research Council for a term of three years beginning July 1, 1920. Professor Veblen's Cambridge Colloquium Lectures on Analysis Situs will be published by the Society late in the fall. Committees were appointed to confer with a committee of the Mathematical Association as to joint plans for future meetings and to prepare nominations for officers and other members of the Council to be elected at the annual meeting in December.

On the recommendation of the Council it was unanimously voted to incorporate the Society under the membership corporations law of the state of New York. The new form of organization will involve hardly any changes beyond those necessary to comply with legal requirements. The Council will continue as at present constituted. But it is necessary under the law to have an elected Board of Trustees; this will consist of the officers and other members of the Council who are elected by the Society at the annual meetings. Otherwise the Constitution and By-Laws, which have come down almost from the beginnings of the Society and which are a highly efficient instrument of government, well worthy of study, will remain practically as at present.

The Council received a preliminary report from the committee on reorganization, which will make specific recommendations at a later meeting. A report was also received

from the committee on the International Mathematical Union, and the formation of an American Section of the Union was approved. The recommendation of the committee on bibliography that a journal of mathematical abstracts be established was approved and the committee was authorized to take steps toward securing the necessary financial support.

The greater part of the afternoon session was devoted to a symposium on the subject of relativity, at which the following papers were presented:

I. "The physical and philosophical significance of the principle of relativity and Einstein's theory of gravitation," by Professor LEIGH PAGE, of Yale University.

II. "Geometric aspects of the Einstein theory," by Professor L. P. EISENHART, Princeton University.

Over fifty members took luncheon together between the sessions, and thirty gathered at the dinner after the meeting.

The regular programme consisted of the following papers:

(1) Professor N. A. COURT: "On a pencil of nodal cubics. Second paper."

(2) Mr. E. L. POST: "Introduction to a general theory of elementary propositions."

(3) Mr. E. L. POST: "Determination of all closed systems of truth tables."

(4) Mr. JESSE DOUGLAS: "The dual of area and of volume."

(5) Professor J. K. WHITTEMORE: "Reciprocity in a problem of relative maxima and minima."

(6) Dr. I. A. BARNETT: "Linear partial differential equations with a continuous infinitude of variables."

(7) Dr. I. A. BARNETT: "Functionals invariant under one-parameter continuous groups in the space of continuous functions."

(8) Professor T. R. HOLLCROFT: "A classification of plane involutions of order four."

(9) Dr. TOBIAS DANTZIG: "A group of line-to-line transformations."

(10) Dr. A. R. SCHWEITZER: "On the iterative properties of the abstract field."

(11) Dr. J. F. RITT: "On the conformal mapping of a region into a part of itself."

(12) Dr. L. R. FORD: "A theorem relative to rational approximations to irrational complex numbers."

(13) Professor L. E. DICKSON: "Report on recent progress in the theory of numbers."

(14) Professor G. D. BIRKHOFF: "Note on the ordinary linear differential equation of the second order."

(15) Professor JOSEPH LIPKA: "The motion of a particle on a surface under any positional forces."

(16) Professor JOSEPH LIPKA: "Note on velocity systems in a general curved space of n dimensions."

(17) Professor J. E. ROWE: "Testing the legitimacy of empirical equations by an analytical method."

(18) Professor OSWALD VEBLEN: "Relations between certain matrices used in analysis situs."

(19) Professor O. D. KELLOGG: "A simple proof of a closure theorem for orthogonal function sets."

(20) Professor C. L. E. MOORE: "Rotation surfaces of constant curvature in a space of four dimensions."

(21) Mr. H. S. VANDIVER: "On Kummer's memoir of 1857 concerning Fermat's last theorem."

(22) Professor NILOS SAKELLARIOU: "A note on the theory of flexion."

Mr. Post was introduced by Professor Keyser; Mr. Douglas by Professor Kasner. The papers of Professor Court, Mr. Post (second paper), Dr. Barnett (first paper), Dr. Dantzig, Dr. Schweitzer, Dr. Ritt, Professor Birkhoff, Professor Lipka, Professor Rowe, Professor Veblen, Professor Kellogg, Professor Moore, Mr. Vandiver, and Professor Sakellariou were read by title.

Abstracts of the papers follow below, being numbered to correspond to the titles in the list above.

1. Continuing the discussion started in his first paper (see this BULLETIN, February, 1920) Professor Altshiller-Court proves that: Two nodal cubics having in common three collinear points, the double point, and the two tangents at this point, are homological, the double point and the base of the three collinear points being the center and the axis of homology, respectively. Several additional properties of the pencil of nodal cubics referred to are derived from this proposition. As an interesting special case of the dual of this proposition is pointed out the following: Two tricuspidal hypocycloids tangent to three concurrent lines are similar and similarly placed, the point of intersection of the three common tangents being the homothetic center of the two curves.

2. In this paper Mr. Post studies in its entirety the deductive system which Whitehead and Russell have developed in Part I, Section A, of their Principia Mathematica. Through the concept of the truth table of a truth function, a uniform method is given for telling whether the assertion of a given propositional function of the system can or cannot be derived from the postulates. By means of this result, a number of properties of the system are obtained, among which is the theorem that any propositional function of the system can either be asserted by means of the postulates or else is inconsistent with them.

Two modes of generalizing the system are considered. One consists in generalizing the primitive functions by means of the truth table concept, and connects up with the work of Sheffer and Nicod. The second or postulational method of generalization is shown to introduce new logical systems.

3. Corresponding to each of the 2^n sets of truth values of the arguments of a truth function $f(p_1, p_2, \cdots, p_n)$, there is a unique truth value of the function. The relation thus set up may be called the truth table of f. Mr. Post considers the systems of truth tables that can be generated by combining arbitrary primitive truth tables and shows that there are 66 different systems generated by primitive tables with no more than three arguments, and 8 infinite families of systems which require tables of four or more arguments. A formula is given for the tables in each system, and it is shown that they include all closed systems of truth tables. These results are applied to the determination of all the ways in which the logical system of truth functions may be generated by independent primitive functions.

4. The object of Mr. Douglas' paper is to develop for the geometry based on the (oriented) line as element a concept analogous to that of area for the geometry based on the point. The main results are as follows:

Denoting by ω, p Hessian line coordinates, the numerical measure of any continuous "mass" of (oriented) lines is equal to $\int\int d\omega dp$. The justification of this definition consists in the fact that the measure so defined possesses the two essential properties of additivity and independence of rigid motion.

The boundary lines of any continuous "mass" envelop, in the usual case, an ordinary closed curve. The double integral $\int\int d\omega dp$ is equal to $\int p d\omega$ taken over this boundary, which last is equal to the length of the bounding curve. The two geometric magnitudes customarily associated with any closed curve, namely the area and the perimeter, thus appear as duals of one another.

By means of a representation of the lines of the plane upon the points of a cylinder as follows: the (oriented) line of Hessian coordinates ω, p corresponds to the point of longitude ω and altitude p, the notion of measure is then extended to non-continuous line sets.

In the latter part of the paper the analogous ideas for three-dimensional space are considered. The measure of a "mass" of planes is found to be $\int\int\int \cos\varphi \, d\theta \, d\varphi \, dp$, where θ, φ, p are polar plane coordinates. On interpreting this integral geometrically, the plane measure of a polyhedron turns out to be equal to $\frac{1}{2}\Sigma e_i A_i$, where e_i is the length of any edge and A_i the dihedral angle of the (oriented) planes meeting in that edge.

5. Professor Whittemore's paper considers the following questions: If $\varphi(x, y) = u$ and $\psi(x, y) = v$ are single-valued real functions of two real variables x, y, having continuous partial derivatives of the first and second orders in the neighborhood of $P(x_0, y_0)$, A: When is $\varphi(x_0, y_0) = u_0$ a maximum or a minimum of u with the condition $\psi = v_0$? B: When is v_0 a maximum or a minimum of v with the condition $\varphi = u_0$? If there are extremes in both A and B, when are they like extremes?

It is proved for the general case, where not both first derivatives of either φ or ψ vanish at P, that a necessary condition for an extreme for both A and B is that the two curves $\varphi = u_0$ and $\psi = v_0$ be tangent at P; that a sufficient condition for an extreme in both problems is that these curves do not osculate at P; when both these conditions are satisfied that the extremes in A and B are like or unlike as φ_v and ψ_v have unlike or like signs at P.

The exceptional case, where both first derivatives of φ or of ψ or of both functions vanish at P, is considered; necessary and sufficient conditions for a maximum or a minimum are obtained in so far as this is possible by use of the first and second deriva-

tives. In some cases the discussion fails to establish such conditions. For several cases it is shown that, in A, u_0 is a maximum or minimum as $\psi_{yy}H + 2\varphi_{yy}\Delta\psi$ is negative or positive at P, where

$$H = \varphi_{xx}\psi_{yy} + \psi_{xx}\varphi_{yy} - 2\varphi_{xy}\psi_{xy}, \qquad \Delta\psi = \psi_{xy}^2 - \psi_{xx}\psi_{yy}.$$

In several of the cases where u_0 and v_0 are extremes in A and B respectively it is proved that they are like or unlike extremes as H is positive or negative.

The invariant character of the conditions established for any change of variables, not singular at P, is considered. Simple examples illustrating the theory are given.

6. In a previous paper Dr. Barnett has given an existence theorem for solutions of linear non-homogeneous partial differential equations with a continuous infinitude of unknowns. In the present paper the theory there developed is applied to linear homogeneous equations. It is found convenient to use the theory of implicit functional equations as developed by Lamson in his Chicago dissertation.

7. Dr. Barnett considers one-parameter continuous groups of transformations $\bar{u}(x) = F[x, u(\xi), t]$ where F is a functional operation yielding for each continuous function of a given range and for each value of the parameter t, another continuous function. Infinitesimal transformations are defined and invariants of the groups are considered. These invariant functionals $G[u(x)]$ are found to be solutions of certain functional equations given in the preceding paper. Examples which are analogues of some of the well-known groups in n-space are studied.

8. Professor Hollcroft discusses the conditions imposed upon two planes when an algebraic (1,4) point correspondence is established between them. The particular correspondence given is defined by a curve of order n with a point P of multiplicity $n - 4$ and a line pencil on P, and is one of ten independent types of plane involutions of order four. An independent type is one that cannot be reduced to another by any birational transformation.

9. In a previous communication to the Society (April, 1916,

Chicago Section) Dr. Dantzig considered certain properties of point-to-point transformations and particularly a certain system of ∞^2 conics which he called the indicating system of the transformation. These properties are here extended to line-to-line transformations, and a special study is made of those transformations which admit a focal line, i. e., a bearer of an invariant range of points. Especially interesting is the case where the line at infinity is a focal line. The indicating system then consists of parabolas. These transformations form a group, the parabolic group G. The fundamental property of a transformation belonging to G is the following: Let l and $\bar{l}$ be two corresponding lines meeting in a point λ; let moreover τ and $\bar{\tau}$ be the points where the indicatrix I touches l and $\bar{l}$ respectively. If c and c' are any two curves touching l in p and p', while their transforms $\bar{c}$ and $\bar{c}'$ touch $\bar{l}$ in $\bar{p}$ and $\bar{p}'$, then the ratio

$$\kappa = \frac{\overline{pp'}}{pp'} = \frac{\overline{\tau\lambda}}{\tau\lambda}$$

and consequently is invariant along the line l. The transformations which leave the ratio κ invariant throughout the plane form a subgroup S of G. However, the subclass of S admitting the same value of κ, say κ_0, does not form a subgroup of S, unless $\kappa_0 = 1$. In this latter case we obtain the Scheffers equilong group.

The rest of the paper is taken up by the generalization of these properties to the case where the focal line is of general position and by the interpretation of the corresponding Scheffer's subgroup. There is also given a general determination of the parabolic transformation admitting an axis and a detailed study of the so-called Laguerre group of line-to-line transformations.

10. In a previous paper Dr. Schweitzer constructed a set of postulates (with explicit reference to iterative compositions) for an abstract field in terms of two undefined quasi-transitive functions, $f(x, y)$ $[x - y]$ and $f_1(x, y)$ $[x/y]$ corresponding to postulates for an abstract group in terms of a relation concretely interpreted by $x \cdot y^{-1}$. In another paper, primarily in reference to the abstract field, he considered quasi-transitive functional equations of the type

$$f\{\lambda f(x, y), \mu f(x, z)\} = \nu f(z, y). \tag{1}$$

A solution of (1) for $\lambda(x) = \mu(x)$, $\nu(x) = x$ is

$$f(x, y) = X^{-1}\{c'X(x)/X(y) + c''\}, \quad \lambda(x) = X^{-1}\{X(x) - c''\},$$

where c' and c'' are constants. Second, a solution of (1) for $\lambda(x) = \mu(x) = x$ is $f(x, y) = X^{-1}\{cX(x) - cX(y)\}$, $\nu(x) = X^{-1}\{cX(x)\}$. If the "arbitrary" constant c is interpreted as a parameter, say t, then special cases of the latter function are

$$f_1(x, y, t_1) = (x - y)\cdot t_1, \qquad f_2(x, y, t_2) = (x - y)/t_2,$$

each of which may serve abstractly as a ternary generating relation for an abstract field.*

11. Let a domain composed of a continuum plus its frontier be mapped conformally, in a one-to-one manner, into a part of itself, so that every frontier point of the original domain becomes an interior point. Dr. Ritt shows that, under the transformation, one point and only one point stays fixed. The derivative of the mapping function is less than unity in modulus at the fixed point. If the transformation is iterated, the consequents of every point of the original domain approach the fixed point. For the case of a simply connected domain this theorem has already been given by Julia, without assuming that the transformation is one-to-one. Julia's method, which is entirely different from that used in the present paper, does not admit of extension to a general continuum.

12. Let ω be any irrational complex number, and consider the inequality

$$\left|\omega - \frac{p}{q}\right| < \frac{k}{|q|^2},$$

where k is real and p/q is a rational fraction (i. e., p and q are each of the form $m + ni$, where m and n are real integers). Dr. Ford proves the following theorem: If $k = 1/\sqrt{3}$ there are infinitely many fractions satisfying the given inequality; if $k < 1/\sqrt{3}$ there exists an everywhere dense set of ω's for each of which the inequality holds for only a finite number of fractions.

* Cf. this BULLETIN, June, 1918, p. 428; March, 1915, p. 295; October, 1915, p. 4; October, 1914, p. 28; Nov., 1918, pp. 58, 59.

13. Professor Dickson made a report on the literature on the theory of numbers during the past decade, prepared in accordance with a decision by the members of the Division of Physical Sciences of the National Research Council to prepare reports of recent progress in various branches of astronomy, mathematics, and physics. While most of these reports will probably cover only the past year, it seemed best in the case of a mathematical subject to cover the past decade, especially in view of the present difficulty of obtaining complete references in a field of mathematics. The present task was simplified by the fact that the writer had already taken great pains to cover the literature to date in his History of the Theory of Numbers, now in course of publication. During the past decade there have appeared fully 1,600 papers, notes and books on this subject. Those relating to divisibility and primality have been considered in volume I of the History, and are only cited en masse in this report in connection with the still more recent papers. In the case of Diophantine analysis (the subject of volume II, in press), only the more important recent papers are cited in this report. But there is given a rather complete list of recent papers on the remaining topics, since their appearance in volume III will be delayed at least a year.

14. Professor Birkhoff shows that it is possible to develop the theory of the ordinary linear differential equation of the second order with real independent variable in such wise that the conditions imposed are both necessary and sufficient throughout. In the usual formulation of the theory the conditions used are not of this type.

15. The complete system of trajectories of a particle constrained to move on a surface under any positional forces form a family of ∞^3 curves, one through each point in each direction for each value of the speed. Professor Lipka derives some of the geometric properties of such a system of curves. One of the simplest of these properties may be stated as follows: in each direction through a given point there passes one trajectory which hyperosculates its circle of constant geodesic curvature; the locus of the centers of geodesic curvature of the ∞^1 hyperosculating trajectories obtained by varying the direction through the point is a conic passing through the

given point in the direction of the force acting at that point. If the force is conservative, the complete systems of brachistochrones and catenaries on a surface also form families of ∞^3 curves. Some properties of these systems are derived, and a comparative study of the three systems, trajectories, brachistochrones and catenaries, is made.

16. In a previous paper,* Professor Lipka characterized certain systems of curves, termed velocity systems, in a general curved space of n dimensions, by means of their geometric properties. In the present note, the author points out a dynamical problem which leads to the same system of curves.

17. After the form of an empirical equation is assumed, the one equation of this assumed type which satisfies the data with least error may be obtained. But the question which remains unanswered is this: Have the data been forced to satisfy an equation of this particular type by virtue of the assumed form of the equation, or do they satisfy an equation of this type naturally? In Professor Rowe's paper an analytical method is developed which enables us to ascertain what type of equation the data (which are known to be correct) most naturally satisfy. The theory involved is a very great addition to, and is not in any sense a substitution for, any good judgment that might be used originally in assuming the most probable type of equation.

18. In Professor Veblen's paper, the matrix giving the number of intersections, counted with regard to sign, of a complete set of i-dimensional manifolds in an n-dimensional manifold, M_n, with a complete set of $(n-i)$-dimensional manifolds of M_n is called an intersection matrix. It is shown how to obtain these matrices from the matrices giving the relations between the oriented j-cells and the oriented $(j-1)$-cells on their boundaries by means of algebraic operations.

19. Let $n(x)$ be the set of normal functions arising from a differential equation of the Sturm type with general self-adjoint homogeneous linear boundary conditions. Professor

* "Natural families of curves in a general curved space of n dimensions," *Trans. Amer. Math. Society*, vol. 13, pp. 77–95.

Kellogg's note gives a simple proof that if this set is closed with respect to continuous functions, it is also closed with respect to summable functions which are not null functions. Hilbert and others have shown that they are closed with respect to continuous functions, and they are therefore closed with respect to the broader class. The interest of the note lies rather in the method of proof than in the results, which are largely already known.

20. In this note Professor Moore discusses the forms of curves that will generate surfaces of constant curvature when rotated by the special rotations leaving a doubly infinite number of planes invariant.

21. In an article in the *Mathematische Abhandlungen* of the Berlin Academy for the year 1857, pages 41–74, Kummer essayed to prove that the relation

$$x^p + y^p + z^p = 0 \tag{1}$$

could not be satisfied in integers, when p is an odd prime not satisfying three given conditions. Based on this result, the conclusion that (1) is impossible for all p's less than 100 was derived by him. In the present paper Mr. Vandiver points out that Kummer made several errors in his argument, which vitiate his results. The paper will appear in the *Proceedings of the National Academy of Sciences.*

F. N. COLE,
Secretary.

STIELTJES DERIVATIVES.

BY PROFESSOR P. J. DANIELL.

THE fundamental theorem for the derivative with respect to a function of limited variation is difficult to prove in the case of several dimensions, and no attempt is made here to consider the most general derived numbers. In place of the method used by the author for one dimension we shall use methods and ideas due to C. de la Vallée Poussin and W. H. Young.*

* C. de la Vallée Poussin, Intégrales de Lebesgue. Paris, 1916, pp. 61–73. W. H. Young, *Proc. London Math. Society* (1914), vol. 13, p. 109. P. J. Daniell, *Transactions Amer. Math. Society* (1918), vol. 19, p. 353.

The theorem to be proved in this paper is:

If in a fundamental interval of several dimensions $\beta(e)$, $F(e)$ are additive functions of sets, $\beta(e)$ non-negative and $F(e)$ absolutely continuous with respect to $\beta(e)$, then there exists a derivative $D_\beta F$ of F with respect to β everywhere except on a set e for which $\beta(e) = 0$; and if E is any set (measurable Borel) contained in the interval,

$$F(E) = \int_E (D_\beta F) d\beta(e).$$

The derivative used is not that obtained from sequences of intervals independent for each point, but a net-derivative.

General considerations. The points p are specified by n coordinates x_r $(r = 1, 2, \cdots, n)$ and an interval consists of a collection of points such that each coordinate lies in a linear continuous interval. In a one-to-one transformation which leaves relative order unchanged, $\beta(e)$, $F(e)$ and $D_\beta F$ will be absolutely invariant, so that the given interval, whatever it may be, can be transformed into one which lies "strictly" within the interval $x_r = 0$ to 1 $(r = 1, 2, \cdots, n)$. By defining $\beta(e)$, $F(e)$ to be 0 for sets not in the transformed interval, we can finally use for the fundamental interval one closed on the left, open on the right, namely

$$0 \leqq x_r < 1 \qquad (r = 1, 2, \cdots, n).$$

$\beta(e)$, $F(e)$ are supposed to be additive in the sense of C. de la Vallée Poussin, that is to say, additive for a countably infinite as well as for a finite number of sets without common points.

Definition 1. If $\beta(e)$, $F(e)$ are additive functions of sets, $F(e)$ is said to be absolutely continuous with respect to $\beta(e)$, if $F(e) = 0$ for all sets e for which $\beta(e) = 0$.

Divide the interval $0 \leqq x_r < 1$ $(r = 1, 2, \cdots, n)$ into 2^{ni} equal subintervals (of order i). These will be of the type

$$(m_r - 1)2^{-i} \leqq x_r < m_r 2^{-i} \qquad (r = 1, 2, \cdots, n),$$

where, for each r, m_r is an integer between 1 and 2^i inclusive. Define the function

$$f_i(p) = F(e)/\beta(e),$$

where e is the interval of order i to which p belongs. Throughout the paper we make the convention that when $\beta(e) = 0$, and consequently $F(e) = 0$, the value 0 shall be assigned to

the meaningless symbol $F(e)/\beta(e)$. Let $h_i(p)$ be, for each p, the upper bound of $f_{i+t}(p)$, and $h(p)$ the limit of the monotone sequence $h_i(p)$. Then $h(p)$ is the upper limit of the sequence $f_i(p)$.

Definition 2. The upper limit $h(p)$ and the corresponding lower limit $g(p)$ are called the upper and lower net-derivatives of $F(e)$ with respect to $\beta(e)$.

Fundamental lemma. If on a set e measurable Borel $h(p) \geqq s$, then

$$F(e) \geqq s\beta(e).$$

We use the notation $e(f, s)$ to denote the set of points for which $f(p) > s$. $f_i(p)$ is constant over each of the intervals of order i, so that $e_i = e(f_i, s)$ consists of a set of intervals each of which satisfies the inequality above, i.e., $F(e_i) \geqq s\beta(e_i)$. Let $e_{i,t} = e_i + e_{i+1} + \cdots + e_{i+t-1}$. Then if the inequality is true for $e_{i,t}$ it is true for $e_{i,t+1}$. We may write $e_{i,t+1} = e_{i,t} + e'$, where $e' = e_{i+t} \cdot Ce_{i,t}$. $Ce_{i,t}$ can be divided into a finite number of intervals of order $i+t$, and all of these which belong to e_{i+t} satisfy the inequality. But $e_{i,t}$ and e' have no common point, so that

$$F(e_{i,t+1}) - s\beta(e_{i,t+1}) = F(e_{i,t}) - s\beta(e_{i,t}) + F(e') - s\beta(e') \geqq 0.$$

By successive induction the inequality is proved for all $e_{i,t}$. If $E_i = \lim e_{i,t}$ as $t \doteq \infty$, since $F(e)$, $\beta(e)$ are additive, $F(E_i) = \lim F(e_{i,t})$, $\beta(E_i) = \lim \beta(e_{i,t})$ and $F(E_i) \geqq s\beta(E_i)$. Now $E_i = e(h_i, s)$; for if at a point p, $h_i > s$ then for some t $f_{i+t} > s$ and p belongs to E_i; while if $h_i(p) \leqq s$, $f_{i+t} \leqq s$ for all t and p belongs to CE_i.

Let $R(s)$ be the set common to all $E_i(s)$ $(i = 1, 2, \cdots)$ and $D(s) = \lim R(s - \epsilon)$ as $\epsilon \doteq 0$, $\epsilon > 0$; then $D(s)$ is the set of points for which $h(p) \geqq s$. For if, at p, $h \geqq s$, then for any $\epsilon > 0$, $h_i > s - \epsilon$ for all i; consequently p belongs to $R(s - \epsilon)$ for any ϵ and therefore to $D(s)$. On the contrary if $h < s$, there is some $\epsilon > 0$ (p fixed) such that $h < s - 2\epsilon$ and a number n can be found so that if $i \geqq n$, $h_i < h + \epsilon < s - \epsilon$. This point p does not belong to $R(s - \epsilon)$ and cannot belong to $D(s)$. But

$$\begin{aligned} F[R(s-\epsilon)] &= \lim F[E_i(s-\epsilon)] \text{ as } i \doteq \infty, \\ &\geqq (s-\epsilon) \lim \beta[E_i(s-\epsilon)] = (s-\epsilon)\beta[R(s-\epsilon)], \\ &\geqq s\beta[R(s)] - \epsilon\beta(I), \end{aligned}$$

where I is the fundamental interval, since $\beta(e)$ is non-negative. Then in the limit as $\epsilon \doteq 0$, $F(D) \geqq s\beta(D)$. When the original interval is replaced by any sub-interval of the type considered the method of reasoning and the function h are unchanged, so that if e is any such interval or finite sum of such intervals without common points

$$F(eD) \geqq s\beta(eD).$$

The inequality will be maintained at successive summations (without common points) and limiting processes. But any set measurable Borel can be obtained from the meshes of the net by these processes, so that if e is measurable Borel, $F(eD) \geqq s\beta(eD)$. This is the required lemma with only a difference in expression.

Conclusion. We are now in a position to prove the main theorem. If B is a set contained in D, e any set $eB = eB \cdot D$. In particular the set B where $h(p) < t$, i.e., where the lower limit of $-f_i(p) > -t$, is included in the set where the upper limit of $-f_i(p) \geqq -t$. On putting $-F$ in place of F and therefore $-f_i$ in place of f_i, the lemma shows that $-F(eB) \geqq -t\beta(eB)$, or $F(eB) \leqq t\beta(eB)$. If $A = BD$, i.e., the set where $s \leqq h(p) < t$, combining the results,

$$s\beta(eA) \leqq F(eA) \leqq t\beta(eA).$$

If on a set e, the upper limit of $f_i(p) = h(p) = +\infty$, then $h(p) > s$ for all s and $s\beta(e) \leqq F(e)$ so that $\beta(e) = 0$, and consequently $F(e) = 0$. The same can be proved of the set where $h = -\infty$. Thus $h(p)$ is finite everywhere except on a set of β-measure 0. Also from the above inequality and the definition of the generalized Stieltjes integral, $h(p)$ is summable and if E is any set measurable Borel

$$F(E) = \int_E h(p)d\beta(e).$$

By exactly the same reasoning, if $g(p)$ is the lower limit of $f_i(p)$, $g(p)$ is finite "nearly everywhere (β)," is summable and

$$F(E) = \int_E g(p)d\beta(e).$$

The difference of the two integrals is 0, while $h \geqq g$ so that $h = g$, i.e., the sequence f_i converges to a single limit $f(p)$

$= h(p) = g(p)$, nearly everywhere (β). This limiting function may be called the net-derivative $D_\beta F$. The main theorem is now proved, since $D_\beta F$ exists and is finite nearly everywhere (β) and if E is measurable Borel,

$$F(E) = \int_E D_\beta F d\beta(e).$$

The proof does not depend on the special method of division; it is sufficient if the "meshes" are divided progressively and if any interval is "Borel measurable" using the meshes as a basic family. The net-derivatives obtained from a finite or countably infinite number of different nets will be the same except on a set of β-measure 0. For if two such net-derivatives are f, k then the sets where $f \geqq k$, and where $f < k$ are each measurable Borel and the integral of $f - k$ with respect to β is 0 over each. But the sum of a countably infinite number of sets of zero measure is itself of zero measure. A proof for *all* net-derivatives considered together is lacking but would be desirable. The method of this paper can easily be extended to derivatives in a countably infinite number of dimensions, corresponding to integrals of the same type exhibited by the author.* For this purpose it is convenient, if not necessary, to commence the sub-division in one dimension after another so that a typical mesh of order i would be

$$(m_r - 1)2^{-(i+1-r)} \leqq x_r < m_r 2^{-(i+1-r)} \quad (r = 1, 2, \cdots, i),$$

$$0 \leqq x_r < 1 \quad (r = i+1, i+2, \cdots),$$

where m_r is an integer between 1 and 2^{i+1-r} $(r = 1, 2, \cdots, i)$ inclusive.

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* P. J. Daniell, *Annals of Mathematics* (1919), vol. 21, p. 30.

SHEFFER'S SET OF FIVE POSTULATES FOR BOOLEAN ALGEBRAS IN TERMS OF THE OPERATION "REJECTION" MADE COMPLETELY INDEPENDENT.

BY DR. J. S. TAYLOR.

(Read before the American Mathematical Society December 31, 1919.)

Some time ago Professor L. L. Dines* demonstrated the fact that while Sheffer's† five postulates for Boolean algebras in terms of "rejection" are independent in the ordinary sense that no one of the postulates is implied by the other four, they are not completely independent in the sense defined by Professor E. H. Moore‡ inasmuch as the negative of the first postulate implies the third, fourth, and fifth postulates of the set. It is the purpose of this paper to demonstrate that if the first postulate is replaced by one postulating a minimum of four instead of two distinct elements the resulting set of postulates is a completely independent set.§

Sheffer's five postulates concerning a system $\Sigma(K, |)$ are:

1. *There are at least two elements in* K.

2. *Whenever* a *and* b *are elements of* K, $a \mid b$ *is an element of* K. *Definition:* $a' = a \mid a$.

3. *Whenever* a *and the indicated combinations of* a *are elements of* K, $(a')' = a$.

4. *Whenever* a, b, *and the indicated combinations of* a *and* b *are elements of* K,

$$a \mid (b \mid b') = a'.$$

* L. L. Dines. "Complete existential theory of Sheffer's postulates for Boolean algebras," this BULLETIN, vol. 21 (Jan., 1915), pp. 183–188.

† H. M. Sheffer, "A set of five postulates for Boolean algebras with applications to logical constants," *Transactions Amer. Math. Society*, vol. 14 (1913), pp. 481–488.

‡ E. H. Moore, "Introduction to a form of general analysis," New Haven Mathematical Colloquium, Yale University Press, page 82.

§ A set of m postulates is said to be ordinarily independent if no one of the m postulates is implied by the others. A set of m postulates is said to be completely independent if, and only if, there are no implicational relations existing among the properties defined either by the postulates as they stand or by the negatives of the postulates. For, if the truth or falsity of one postulate implies either that another postulate is true or that it is false, it would seem either that the two postulates are concerned with two aspects of the same fundamental property, or that there are two fundamental properties involved in such a manner that one of the postulates, at least, deals with both properties.

5. *Whenever a, b, c, and the indicated combinations of a, b and c are elements of K,*

$$[a \mid (b \mid c)]' = (b' \mid a) \mid (c' \mid a).$$

As stated above, the negative of the first postulate implies the third, fourth, and fifth postulates. For, if K contains less than two distinct elements, these three postulates are satisfied either evidently or vacuously according as the second postulate does or does not hold. This difficulty is no longer encountered, however, if, instead of assuming postulate 1, we assume the following.

1′. *There are at least four distinct elements in K.*

The truth of this statement is demonstrated by the exhibition of thirty-two systems having the $2^5 = 32$ possible characters $(+++++)$, $(++++-)$, $\cdots$, $(-----)$, the ith sign being plus or minus according as the ith postulate is or is not satisfied.

There are two systems with K^{singular}, eight with K^{dual}, six with K^{triple}, and sixteen with $K^{\text{quadruple}}$. In each case the systems have been chosen with as few elements as possible.

In accordance with custom the result of combining elements is given by means of tables. For example, if K contains two elements l_1 and l_2 and if $l_1 \mid l_1 = l_1$, $l_1 \mid l_2 = l_1$, $l_2 \mid l_1 = l_2$, and $l_2 \mid l_2 = l_2$, this will be stated in the form

$1 \mid$	l_1	l_2
$l_1 \mid$	l_1	l_1
$l_2 \mid$	l_2	l_2

If $l_i \mid l_j$ does not give an element in K, this will be indicated by saying that $l_i \mid l_j = x$. The extension of this sort of table to systems containing more than two elements is obvious.

The thirty-two systems with their indicated characters follow:

K Singular.

System I_1. $(-++++)$ $l_1 \mid l_1 = l_1$.

System I_2. $(--+++)$ $l_1 \mid l_1 \neq l_1$.

K Dual.

System II_1. $(-++++-)$

$1 \mid$	l_1	l_2
$l_1 \mid$	l_1	l_1
$l_2 \mid$	l_2	l_2

System II_2. $(-+++-+)$

$1 \mid$	l_1	l_2
$l_1 \mid$	l_1	l_2
$l_2 \mid$	l_2	l_2

System II_3.

$(-+-++)$

	1	l_1	l_2
l_1	l_1	l_1	l_1
l_2	l_1	l_1	l_1

System II_4.

$(-++--)$

	1	l_1	l_2
l_1	l_1	l_1	l_2
l_2	l_1	l_1	l_2

System II_5.

$(-+--+)$

	1	l_1	l_2
l_1	l_1	l_1	l_2
l_2	l_2	l_2	l_1

System II_6.

$(--+-+)$

	1	l_1	l_2
l_1	l_1	l_1	l_2
l_2	x	l_2	l_2

System II_7.

$(---++)$

	1	l_1	l_2
l_1	l_1	l_1	x
l_2	x	l_1	l_1

System II_8.

$(----+)$

	1	l_1	l_2
l_1	l_1	l_2	l_1
l_2	x	l_2	l_2

K Triple.

System III_1.

$(-+-+-)$

	1	l_1	l_2	l_3
l_1	l_1	l_1	l_2	l_3
l_2	l_1	l_1	l_1	l_1
l_3	l_2	l_1	l_2	l_2

System III_2.

$(--++-)$

	1	l_1	l_2	l_3
l_1	l_1	l_1	l_2	l_3
l_2	l_3	l_3	l_3	l_1
l_3	l_2	x	x	x

System III_3.

$(-+---)$

	1	l_1	l_2	l_3
l_1	l_1	l_1	l_2	l_3
l_2	l_3	l_3	l_1	l_3
l_3	l_2	l_3	l_3	l_2

System III_4.

$(--+--)$

	1	l_1	l_2	l_3
l_1	l_1	l_1	l_2	l_3
l_2	l_1	l_1	l_2	l_3
l_3	l_1	x	l_3	l_3

System III_5.

$(---+-)$

	1	l_1	l_2	l_3
l_1	l_1	l_1	l_2	x
l_2	l_1	l_1	l_1	l_1
l_3	l_2	l_1	l_2	l_2

System III_6.

$(-----)$

	1	l_1	l_2	l_3
l_1	l_1	l_1	l_2	x
l_2	l_3	l_1	l_2	l_2
l_3	x	x	l_2	l_2

K Quadruple.

System IV_1.

(+++++)

1	l_1	l_2	l_3	l_4
l_1	l_2	l_1	l_4	l_3
l_2	l_1	l_1	l_1	l_1
l_3	l_4	l_1	l_4	l_1
l_4	l_3	l_1	l_1	l_3

System IV_2.

(++++−)

1	l_1	l_2	l_3	l_4
l_1	l_1	l_1	l_1	l_1
l_2	l_2	l_2	l_2	l_2
l_3	l_3	l_3	l_3	l_3
l_4	l_4	l_4	l_4	l_4

System IV_3.

(+++−+)

1	l_1	l_2	l_3	l_4
l_1	l_1	l_1	l_1	l_1
l_2	l_1	l_2	l_1	l_1
l_3	l_1	l_1	l_3	l_1
l_4	l_1	l_1	l_1	l_4

System IV_4.

(++−++)

1	l_1	l_2	l_3	l_4
l_1	l_1	l_1	l_1	l_1
l_2	l_1	l_1	l_1	l_1
l_3	l_1	l_1	l_1	l_1
l_4	l_1	l_1	l_1	l_1

System IV_5.

(+−+++)

1	l_1	l_2	l_3	l_4
l_1	x	x	x	x
l_2	x	x	x	x
l_3	x	x	x	x
l_4	x	x	x	x

System IV_6.

(+++−−)

1	l_1	l_2	l_3	l_4
l_1	l_1	l_2	l_3	l_4
l_2	l_1	l_2	l_3	l_4
l_3	l_1	l_2	l_3	l_4
l_4	l_1	l_2	l_3	l_4

System IV_7.

(++−+−)

1	l_1	l_2	l_3	l_4
l_1	l_2	l_2	l_1	l_1
l_2	l_1	l_1	l_1	l_1
l_3	l_1	l_1	l_1	l_1
l_4	l_1	l_1	l_1	l_1

System IV_8.

(+−++−)

1	l_1	l_2	l_3	l_4
l_1	l_1	l_1	x	x
l_2	l_2	l_2	x	x
l_3	x	x	x	x
l_4	x	x	x	x

System IV_9.

(++−−+)

1	l_1	l_2	l_3	l_4
l_1	l_1	l_2	l_1	l_1
l_2	l_2	l_1	l_1	l_1
l_3	l_1	l_1	l_1	l_1
l_4	l_1	l_1	l_1	l_1

System IV_{10}.

(+−+−+)

1	l_1	l_2	l_3	l_4
l_1	l_1	l_2	x	x
l_2	x	l_2	x	x
l_3	x	x	x	x
l_4	x	x	x	x

System IV_{11}.

(+−−++)

1	l_1	l_2	l_3	l_4
l_1	l_1	x	x	x
l_2	x	l_1	x	x
l_3	x	x	x	x
l_4	x	x	x	x

System IV_{12}.

(++−−−)

1	l_1	l_2	l_3	l_4
l_1	l_2	l_1	l_1	l_1
l_2	l_1	l_3	l_1	l_1
l_3	l_3	l_3	l_3	l_3
l_4	l_4	l_4	l_4	l_4

System IV_{13}.

(+−+−−)

1	l_1	l_2	l_3	l_4
l_1	l_1	l_2	x	x
l_2	l_1	l_2	x	x
l_3	x	x	x	x
l_4	x	x	x	x

System IV_{14}.

(+−−+−)

1	l_1	l_2	l_3	l_4
l_1	l_1	l_2	l_3	x
l_2	l_1	l_1	l_1	x
l_3	l_2	l_1	l_2	x
l_4	x	x	x	x

System IV_{15}.

(+−−−+)

1	l_1	l_2	l_3	l_4
l_1	l_2	l_1	x	x
l_2	x	l_2	x	x
l_3	x	x	x	x
l_4	x	x	x	x

System IV_{16}.

(+−−−−)

1	l_1	l_2	l_3	l_4
l_1	l_1	l_2	x	x
l_2	l_3	l_1	l_2	x
l_3	x	x	l_2	x
l_4	x	x	x	x

It may be of interest to note that a similar change in the first postulate of Bernstein's* set of four postulates in terms of the operator "rejection" also makes that set completely independent, as I have shown in an earlier paper,† and that furthermore this same change makes my own set of five postulates in terms of "exception"‡ completely independent (together with a change in the fifth postulate at a sacrifice of simplicity). It would be interesting to ascertain whether this postulation of a minimum of four distinct elements is sufficient to being about the complete independence of any

* B. A. Bernstein, "A set of four independent postulates for Boolean algebras," *Transactions Amer. Math. Society*, vol. 17 (1916), pp. 50–52.

† J. S. Taylor, "Complete existential theory of Bernstein's set of four postulates for Boolean algebras," *Annals of Mathematics*, second series, vol. 19, no. 1 (Sept., 1917), pp. 64–69.

‡ J. S. Taylor, "A set of five postulates for Boolean algebras in terms of the operation 'exception,'" *University of California Publications in Mathematics*, vol. 1 (April 12, 1920), pp. 241–248.

set of postulates for Boolean algebras where the remaining postulates of the set are already free from all implicational relationships among themselves.

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ROTATION SURFACES OF CONSTANT CURVATURE IN SPACE OF FOUR DIMENSIONS.

BY PROFESSOR C. L. E. MOORE.

(Read before the American Mathematical Society April 24, 1920.)

1. IN space of four dimensions there are two special rotations each of which has circles for path curves. In this note I shall discuss the surfaces generated by these special rotations which have constant curvature. The first type is given by the equations

$$X = x\cos t - y\sin t, \quad Y = x\sin t + y\cos t, \tag{1}$$
$$Z = z, \quad W = w.$$

This rotation leaves each point of the zw-plane invariant and any plane completely perpendicular to it is left invariant as a plane but not point for point. The rotation then is simply isomorphic with a rotation in the xy-plane.*

If the curve

$$(c) \qquad x = x(s), \quad y = y(s), \quad z = z(s), \quad w = w(s),$$

where s denotes arc length measured from a fixed point, is rotated, equations (1) are the parametric equations of the surface generated. The parameter curves $s=$ const., $t=$ const. will be orthogonal if

$$(2) \qquad xy' - x'y = 0 \quad \text{or} \quad y = kx,$$

where primes denote derivatives with respect to s. Hence *the meridian curves (orthogonal trajectories of the path curves) on a surface generated by rotation* (1), *lie in a 3-space which contains the absolutely invariant plane.* If the meridian curve

* Phillips and Moore, "Rotations in space of even dimensions," *Proceedings Amer. Academy*, vol. 55.

is a plane curve its plane must lie in one of these 3-spaces and consequently cut the zw-plane in a line. It will then also cut a set of invariant planes which are completely perpendicular to the zw-plane in lines. A plane which cuts the zw-plane in a line will then generate a 3-space by the rotation (1). Hence the surfaces of rotation, generated by (1), for which the meridians are plane curves must lie in a 3-space. Such surfaces of constant curvature are already well known.

If the meridians are not plane curves they must satisfy (2), and for simplicity we will assume that it lies in the xzw-space. The element of arc on such a surface is

$$d\sigma^2 = ds^2 + x^2 dt^2, \tag{3}$$

where s denotes the arc on the meridian curve and σ the arc of any curve traced on the surface of rotation. If the surface has constant positive curvature,*

$$\frac{1}{a^2} = -\frac{1}{x}\frac{d^2x}{dt^2}.$$

The integral of this is

$$x = c\cos(s/a + b). \tag{4}$$

By properly choosing the reference system b can be made to vanish. This we will assume done. We can then write

$$dx^2 + dz^2 + dw^2 = ds^2 \quad \text{or}$$

$$dz^2 + dw^2 = \left(1 - \frac{c^2}{a^2}\sin^2\frac{s}{a}\right)ds^2. \tag{5}$$

For a rotation surface in 3-space equation (5) reduces to

$$dz^2 = \left(1 - \frac{c^2}{a^2}\sin^2\frac{s}{a}\right)ds^2, \tag{6}$$

the solution of which is well known. For the case here considered, $dz^2 + dw^2 = ds^2$ is the square of the element of arc of the projection of the meridian curve on the zw-plane. Equation (6) then becomes

$$ds_1^2 = \left(1 - \frac{c^2}{a^2}\sin^2\frac{s}{a}\right)ds^2.$$

* Eisenhart, Differential Geometry, p. 270.

The solution for s_1 in terms of s is the same as the solution of (6) for z in terms of s. The projection s_1 then bears the same relation to x that z does for a rotation surface of constant positive curvature in 3-space. Hence *to obtain curves in the xzw-space which can be used as meridian curves of a surface of revolution of constant positive curvature, trace on the zw-plane curves* (c) *which will generate such surfaces. Then roll this plane into a cylinder with elements parallel to OX, making OZ roll into the section of the cylinder in the zw-plane. The curves into which the curves* (c) *roll are the curves sought.*

2. The second kind of rotation which has circular path curves is

$$\begin{aligned} X &= x\cos t - y\sin t, \quad Y = x\sin t + y\cos t, \\ Z &= z\cos t - w\sin t, \quad W = z\sin t + w\cos t. \end{aligned} \tag{7}$$

This rotation leaves ∞^2 planes of a linear congruence invariant.* These can be arranged into pairs of completely perpendicular planes. If the curve (c) is rotated (7) will be the parametric equations of the resulting surface. The parameter curves on this surface will be orthogonal if

$$x'y - xy' + z'w - zw' = 0. \tag{8}$$

Now by this rotation† if a line cuts one path curve orthogonally it will cut all orthogonally and consequently must lie in a plane which cuts two completely perpendicular invariant planes in lines. Therefore the solutions of (8) must lie on cones whose tangent planes cut a pair of completely perpendicular invariant planes in lines. This equation shows that no 3-space curve lying in a 3-space passing through the origin can be used as a meridian curve. For by choosing the axes properly such a 3-space can be represented by $x = 0$ and consequently, from (8), $z = kw$ and hence the curve must be plane. If the curve is plane it must lie in a plane cutting two completely perpendicular invariant planes in lines. We will discuss this case first.

Let the curve lie in the xz-plane. Equations (7) then become

$$X = x\cos t, \quad Y = x\sin t, \quad Z = z\cos t, \quad W = z\sin t.$$

* Moore, "Rotations in hyperspace," *Proceedings Amer. Academy*, vol. 53.

† Moore, "Surfaces of rotation in space of four dimensions," *Annals of Math.*, vol. 21.

The element of arc of the surface formed by rotating $x = x(s)$, $z = z(s)$ is

$$d\sigma^2 = ds^2 + (x^2 + z^2)dt^2. \tag{9}$$

If the surface has constant positive curvature, then

$$\rho = \sqrt{x^2 + z^2} = c \cos\frac{s}{a}. \tag{10}$$

Using polar coordinates in the xz-plane, we then have

$$d\rho^2 + \rho^2 d\theta^2 = \frac{a^2 d\rho^2}{c^2 - \rho^2} \quad \text{or} \quad \frac{1}{\rho}\sqrt{\frac{\rho^2 + a^2 - c^2}{c^2 - \rho^2}}\, d\rho = d\theta. \tag{11}$$

Just as in three dimensions, there are three types of curves depending on the relations between a and c.

(a) If $a = c$, the solution is

$$\rho = a \sin(\theta + b).$$

Thus *any circle of radius $a/2$ passing through the origin will rotate, by* (7), *into a surface of constant positive curvature a.* If the circle has its center at the origin, it will generate a developable surface as is at once evident from (9).

(b) If $a < c$, the solution is

$$\frac{1}{2}\sin^{-1}\frac{2\rho^2 + a^2 - 2c^2}{a^2} - \frac{1}{2}\sqrt{\frac{c^2 - a^2}{c^2}}\sin^{-1}\frac{2c^2(a^2 - c^2) + (2c^2 - a^2)\rho^2}{c^2\rho^2} = \theta + b.$$

(c) If $a > c$, the solution is

$$\frac{1}{2}\sin^{-1}\frac{2\rho^2 + a^2 - 2c^2}{a^2} - \frac{1}{2}\sqrt{\frac{a^2 - c^2}{c^2}}\cosh^{-1}\frac{2c^2(a^2 - c^2) + (2c^2 - a^2)\rho^2}{c^2\rho^2} = \theta + b.$$

In cases (a) and (b) the solutions can be written in finite form, while the corresponding solutions for 3-space can be written in parametric form only by the use of elliptic functions.*

* See Bianchi, vol. 1, p. 233.

To determine the form of these curves we return to equation (11) and write it in the form

$$\sqrt{\frac{\rho^2 + a^2 - c^2}{c^2 - \rho^2}} = \frac{\rho d\theta}{d\rho} = \tan \psi,$$

where ψ is the angle between the tangent line and the radius vector.

If $a < c$, this last equation shows that $c^2 - a^2 < \rho^2 < c^2$ and hence the curve lies between two concentric circles of radii $\sqrt{c^2 - a^2}$ and c. Also at the inner circle ψ is zero and at the outer circle $\pi/2$. This curve then consists of segments which start perpendicular to the inner circle, touch the outer circle and return perpendicular to the inner circle. If $a > c$ the curve is in the form of a loop which crosses at the origin and is tangent to the circle $\rho = c$. For given values of a and c equation (11) shows that all curves cut the circle $\rho = \text{const.}$ at the same angle and have the same radius of curvature at the points of intersection. The curves then are symmetrical about the line joining the origin to the point of tangency with $\rho = c$. All curves are obtained by rotating a given one about the origin.

3. If the surface is one of constant negative curvature,

$$\frac{1}{a^2} = \frac{1}{\rho}\frac{d^2\rho}{dt^2}.$$

Hence $\rho = c_1 \cosh s/a + c_2 \sinh s/a$.

The three cases to consider are $c_1 = 0$; $c_2 = 0$; $c_1 = c_2$. The square of the element of arc in each of these three cases is

(i) $$d\sigma^2 = ds^2 + c^2 \cosh^2 s/a \, dt^2,$$

(ii) $$d\sigma^2 = ds^2 + c^2 \sinh^2 s/a \, dt^2,$$

(iii) $$d\sigma^2 = ds^2 + c^2 e^{2s/a} \, dt^2.$$

Any case other than these may be obtained by taking for ρ either of the values $\cosh (s/a + b)$, $\sinh (s/a + b)$ where b is constant. For the first case, following the previous discussion, we have

(12) $$d\rho^2 + \rho^2 d\theta^2 = \frac{a^2 d\rho^2}{\rho^2 - c^2} \quad \text{or} \quad \sqrt{\frac{\rho^2 - c^2 - a^2}{c^2 - \rho^2}} = \frac{\rho d\theta}{d\rho}.$$

In this case also the equation is readily integrated in terms

of elementary functions, but we will obtain the properties of the curve from equation (12) itself. The first thing observed is that $c^2 < \rho^2 < c^2 + a^2$. Hence the curve does not pass through the origin for any value of c but lies between two concentric circles with center at the origin as before. In this case however the curve is tangent to the inner circle and perpendicular to the outer circle. This corresponds to the pseudospherical surface of hyperbolic type.

In case (ii) the differential equation is

$$\sqrt{\frac{a^2 - c^2 - \rho^2}{c^2 + \rho^2}} = \frac{\rho d\theta}{d\rho}.$$

In this case, then, $a > c$ and $\rho^2 < a^2 - c^2$. The curve then lies inside the circle $\rho = \sqrt{a^2 - c^2}$ to which it is perpendicular. As the value of ρ decreases, the value of ψ approaches $\tan^{-1}\sqrt{(a^2 - c^2)/a^2}$ and therefore the curve approaches the origin, cutting all the radius vectors at the above angle.

In case (iii) we have

$$\frac{1}{\rho}\sqrt{a^2 - \rho^2} = \frac{\rho d\theta}{d\rho}.$$

The curve then lies inside the circle $\rho = a$ which it cuts perpendicularly. As ρ decreases, ψ approaches $\pi/2$. This curve then approaches the origin, cutting the radius vectors at right angles. This is the limiting case of either of the above. If any of the above curves are rotated about the origin in the xz-plane, when rotated by (7) they will still generate a surface of constant negative curvature. The forms of all these differentials lend themselves readily to graphical solution so that a picture of the curves is easily obtained.

4. In case of the general rotation (7) the square of the element of arc is

$$d\sigma^2 = ds^2 + (x^2 + y^2 + z^2 + w^2)dt^2 \tag{13}$$

and if the surface is one of constant positive curvature,

$$\rho = \sqrt{x^2 + y^2 + z^2 + w^2} = c\cos\frac{s}{a}. \tag{14}$$

Consequently the distance ρ of the points from the origin is a function of s. Then if we have a curve (c) satisfying

equation (14) and unroll the cone which projects this curve from the origin, on the xz-plane, the curve into which (c) goes will satisfy (10) and hence will rotate into a surface of constant positive curvature. Hence *the curves of 4-space which rotate by* (7) *into surfaces of constant positive curvature are obtained by tracing on the xz-plane curves* (c) *which rotate by* (7) *into surfaces of constant positive curvature, and then rolling the xz-plane into a cone with vertex at the origin. The curves into which the curves* (c) *go are those sought.* Of course to obtain curves which cut the path curves orthogonally the above cones must be such that each of their tangent planes cuts a pair of completely perpendicular planes in lines. Curves which generate surfaces of constant negative curvature are obtained in a similar manner.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

SHORTER NOTICES.

The Mystery of Space. A study of the hyperspace movement in the light of the evolution of new psychic faculties, and an inquiry into the genesis and essential nature of space. By ROBERT T. BROWNE. New York, E. P. Dutton and Company, 1919. 8vo. 395 pp. $4.00.

THIS book is a mixture of cosmogony, psychology, and geometry; heralded in a recent flyer as "an epoch-making work."

With the cosmogony and psychology we can have little to do. Anyone obviously has a perfect right to philosophize about the universe and the true nature of space as much as he pleases and to dress his philosophy in Greek nomenclature to give it a scientific aspect if he chooses. But the reader may be pardoned if he is tempted to compare the periodic wanderings of Mr. Browne's "monopyknon," from chaos through seven stages of "pyknosis" before emerging into physical being, and thence through sentient, mental, and spiritual stages back to chaos again, with Goethe's story of the Homunculus; and to assert that the one is as mediaeval in character as the other was intended to be. Nor can one deny to the author the right to hold and to defend, if he can, the theory that humanity will one day develop to the point where,

through the agency of the pineal gland and the pituitary body, intuition will supersede intellect and mathematics will cease to be useful or, if needed, will be known intuitively to the infants of that millennium. If a common mortal finds difficulty in following the arguments in support of this theory, it is probably because he has regarded his pineal gland and pituitary body, if conscious of them at all, as rudimentary rather than embryonic, and so has failed to cultivate their latent powers, as Mr. Browne would have us believe clairvoyants, and their like, do.

But with geometry the case is somewhat different. The author's profound apology to mathematicians for his temerity in entering this field indicates that he was conscious of his lack of training for the work he essayed to do. That he attempted to prepare for it by reading about non-euclidean geometry and hyperspace, is shown by a more or less detailed reproduction of the history of non-euclidean geometry from the earliest attempts to prove Euclid's parallel postulate to the present time; by numerous quotations from various writers on non-euclidean geometry and hyperspace; and by an appendix containing a bibliography of over one hundred articles and books. That he failed to digest this material is shown by the persistent confusion of non-euclidean geometry with hyperspace, and by his supposition that these subjects were invented by mathematicians for the purpose of explaining the real nature of space. Of course he must refute these spurious explanations in order to introduce his own idea of space. Naturally he can offer no proof that real space is not non-euclidean in character or that it has exactly three dimensions, for he scorns basic axioms and takes mathematicians to task for deifying the definition. One wonders why he did not seize upon hyperbolic geometry as a coup de maître in support of his theory that real space is finite, limited, and surrounded by chaos, or "spacelessness." The best he can do, however, is to assert the absurdity of all geometrical conceptions, other than the classical euclidean, styling them "superfœtated hypotheses" or "mathetic divertisements." It is apparently an easy and interesting game for Mr. Browne to tilt rhetorical lances at non-euclidean geometry and the fourth dimension and he indulges frequently in this sport throughout the book. But this is a dangerous game for one who apologizes upon entering it and exhibits throughout a lack of knowledge of

the objects at which he aims his lances, and our author must not expect to fare better at it than did his prototype who mistook the windmills for something other than they were.

L. WAYLAND DOWLING.

Junior High School Mathematics, Books 1 and 2. By E. H. TAYLOR and FISKE ALLEN. New York, Henry Holt and Company, 1919.

To one who has followed the trend of mathematical education during the last ten or fifteen years, it is apparent that the voluminous discussion on coordination of elementary mathematics is beginning to bear fruit. Recent elementary texts like the one under discussion show very clearly that teachers and authors now have a definite aim in view. The two most notable features of these modern texts are, first, the early introduction of geometric ideas and constructions without formal demonstrations, and second, the socializing of elementary mathematics by interesting the pupil in everyday activities which require computations and geometric constructions.

The text under review is a modern coordinated treatment of arithmetic, algebra and geometry for use in the seventh and eighth grades. Book 1 contains first a review of the fundamental operations of arithmetic with numerous problems in keeping accounts and simple business transactions. This is followed by a study of the formula as an introduction to literal arithmetic and algebra; percentage and its simpler practical applications; measurement of lines and angles; triangles and other constructions with ruler and compasses; parallel lines, quadrilaterals and polygons; and the mensuration of areas. The arrangement of subject matter is excellent, and the problem material covers a very extensive range of useful applications.

Book 2 continues the three lines of work begun in the first volume, viz., arithmetic, algebra and geometry. Algebra is approached through the formula and the use of letters to shorten statements in words; the square root of numbers is found arithmetically and also graphically by using the Pythagorean theorem; ratio and proportion are also treated both arithmetically and geometrically; this is followed by the mensuration of simple solids with numerous concrete problems; explanation of the negative; problems involving simple

equations; graphs and their application to statistical data; and the technical applications of percentage, such as banking and investment.

The book closes with a few extended problems which follow some special activity involving mathematical calculations through all its branches. A typical problem is the farm project, which includes financing the purchase and equipment of the farm; the construction of farm buildings; the purchase of live stock and implements; various agricultural problems; and finally the calculation of expenses and returns on the investment. Such problem material provides the most effective form of coordination and also instills valuable lessons in thrift and industry, so much needed at the present time.

Such books as these mark a distinct advance in the teaching of elementary mathematics, and cannot fail to make teaching more efficient and lead to a better appreciation of what mathematics is, and of its importance as a factor in common school education.

S. E. SLOCUM.

Tables from the Mathematical Theory of Investment. By E. B. SKINNER. Boston, Ginn and Company, 1917. 26 pages. Price 36 cents.

Problems in the Mathematical Theory of Investment. By G. R. CLEMENTS. Boston, Ginn and Company, 1917. 24 pages. Price 32 cents.

THESE tables are reprinted without change or introductory explanation from Professor Skinner's Mathematical Theory of Investment. In most cases, however, the descriptive titles of the tables are sufficient to make the use of them clear to persons who have not read the book from which they are reprinted.

Professor Skinner's textbook, first published in 1913, has been widely adopted and is generally recognized as the best American textbook on the subject. In class exercises and in final examinations it is frequently convenient to have the students use the tables without using the text. The printing of the tables separately was therefore highly desirable.

The collection of problems by G. R. Clements contains one hundred examples of the same general nature as those in Professor Skinner's text. The problems are not, however,

graded by subjects, so that the student is required to determine for himself the class into which each problem falls and to select the formula required for its solution. The problems are preceded by three pages of hints as to how to analyze and solve a problem. The more important formulas used are listed. Hints are given in connection with some of the more difficult examples. A complete set of answers is printed at the end of the book.

This collection will be of great assistance to teachers who wish a supplementary list of miscellaneous examples for review, or who wish to use the textbook for a series of years without the necessity of assigning the same problems year after year.

CLINTON H. CURRIER.

NOTES.

PROFESSORS L. E. DICKSON, of the University of Chicago, and L. P. EISENHART, of Princeton University, have been elected delegates of the American section of the International mathematical union to attend the meeting of the union at the University of Strasbourg beginning September 18, 1920.

THE April number (volume 42, number 2) of the *American Journal of Mathematics* contains the following papers: "On the convergence of certain classes of series of functions," by R. D. CARMICHAEL; "On the solution of linear equations in infinitely many variables by successive approximations," by J. L. WALSH; "Self-dual plane curves of the fourth order," by L. E. WEAR; "On the groups of isomorphisms of a system of abelian groups of order p^m and type $(n, 1, 1, \cdots, 1)$," by L. C. MATHEWSON; "On the satellite line of the cubic," by R. M. WINGER.

AT the meeting of the Edinburgh mathematical society on May 14, the following papers were read: By E. T. WHITTAKER, "Some developments in curve fitting and the calculus of differences"; by C. G. KNOTT, "Robert Hooke on molecular interplay." At the meeting of June 11, a paper was read by

G. SMEAL, "On direct and inverse interpolation by divided differences."

BEGINNING with volume 81, the *Mathematische Annalen* will be published by Julius Springer, of Berlin, instead of by Teubner as heretofore.

THE following university courses in mathematics are announced for the academic year 1920–1921:

PRINCETON UNIVERSITY. By Professor H. B. FINE: Theory of functions, three hours.—By Professors L. P. EISENHART and OSWALD VEBLEN: Seminar in relativity, three hours.—By Professor OSWALD VEBLEN: Projective geometry, three hours.—By Professor J. H. M. WEDDERBURN: Theory of functions of a real variable, three hours.—By Professor J. W. ALEXANDER: Conformal representation, three hours.

STANFORD UNIVERSITY. First quarter (October–December):—By Professor R. E. ALLARDICE: Advanced calculus, five hours.—By Professor H. F. BLICHFELDT: Vector analysis, four hours. Second quarter (January–March):—By Professor GREEN: Modern coordinate geometry, five hours.—By Professor R. E. ALLARDICE: Projective geometry, five hours.—By Professor H. F. BLICHFELDT: Differential equations, five hours. Third quarter (April–June):—By Professor GREEN: Modern coordinate geometry, five hours.—By Professor R. E. ALLARDICE: Theory of functions, five hours.—By Professor H. F. BLICHFELDT: Non-euclidean geometry, four hours.

THE Buchhandlung Gustav Fock, of Leipzig, offers for sale the library of the late Professor MORITZ CANTOR, of Heidelberg, the historian of mathematics. The library consists of about 2,000 volumes and 2,500 pamphlets.

AT the University of Berlin, Professor L. E. BROUWER, of the University of Amsterdam, has been appointed professor of mathematics, Professor R. VON MISES, of the Dresden technical school, has been appointed professor of applied mathematics, and Dr. ISSAI SCHUR has been promoted to a full professorship of mathematics. Professor C. CARATHÉODORY has resigned, to accept a professorship at the National University at Athens.

AT the University of Clermont, Dr. G. GIRAUD has been appointed chargé de cours for the differential and integral calculus, in place of Professor A. C. E. PELLET, who has retired from active teaching.

DR. RENÉ GARNIER has been appointed chargé de cours for theoretical and applied mechanics at the University of Poitiers.

AT the University of Nancy, Dr. LÉOPOLD LEAU has been appointed professor of the differential and integral calculus, as successor to Dr. A. S. E. HUSSON.

DR. L. SIRE, of the University of Rennes, has been appointed professor of applied mathematics at the University of Lyons, as successor to the late Professor D. J. B. FLAMME.

PROFESSOR H. LAMB, of the University of Manchester, Sir T. L. HEATH, and Professor W. H. BRAGG, of the University of London, have been elected honorary fellows of Trinity College, Cambridge.

THE senate of the University of Dublin has conferred the honorary degree of doctor of science on Professor W. H. BRAGG, of the University of London, and Professor R. A. MILLIKAN, of the University of Chicago.

MR. W. E. H. BERWICK has been appointed lecturer in mathematics at the University of Leeds.

PROFESSOR JACQUES HADAMARD has been elected a foreign honorary member of the American academy of arts and sciences.

PROFESSOR AUGUSTUS TROWBRIDGE, of the department of physics of Princeton University, has been appointed chairman of the division of astronomy, mathematics and physics of the National research council, for the year beginning July 1, 1920.

AMONG the grants for research made by the American association for the advancement of science in April are three hundred dollars to Professor S. LEFSCHETZ, of the University

of Kansas, to assist in the publication of his memoir on algebraic surfaces, which was awarded the Bordin prize by the Paris academy of sciences, and one hundred dollars to Professor OLIVE C. HAZLETT, of Mount Holyoke College, in support of her work in the theory of hypercomplex numbers and invariants.

ON the recommendation of the National academy of sciences, Columbia University has awarded its Barnard medal for meritorious service to science to Professor ALBERT EINSTEIN, for "his highly original and fruitful development of the fundamental concepts of physics through the application of mathematics."

DR. R. S. WOODWARD will retire from the presidency of the Carnegie Institution of Washington at the end of the present year, after sixteen years of service.

DR. J. W. ALEXANDER has been appointed assistant professor of mathematics at Princeton University.

IN the department of mathematics at the University of Minnesota, associate professor W. H. BUSSEY has been promoted to a full professorship and appointed assistant dean of the college of science, literature and the arts, and assistant professors A. L. UNDERHILL, R. W. BRINK, and W. L. HART have been promoted to associate professorships. Mr. F. K. BAIER, JR., of Pennsylvania State College, and Dr. GLADYS GIBBENS, of the University of Chicago, have been appointed instructors.

ASSOCIATE professor J. E. ROWE, of the Pennsylvania State College, was promoted to a full professorship of mathematics in January, 1920.

ASSISTANT professor C. W. WESTER, of Iowa State Teachers' College, has been promoted to a full professorship of mathematics.

AT Oberlin College, associate professor W. D. CAIRNS has been made full professor and head of the department of mathematics. Professor FREDERICK ANDEREGG has retired from active teaching, after thirty-three years of service.

DR. J. R. MUSSELMAN, of Washington University, has been appointed associate in mathematics at Johns Hopkins University.

MR. JESSE DOUGLAS has been appointed instructor in mathematics at Columbia University.

AT the Carnegie Institute of Technology, assistant professor J. R. EVERETT, of Baker University, and Professor G. W. HESS, of Bethany College, have been appointed instructors in mathematics.

ASSISTANT professor A. E. BABBITT, of the University of Nebraska, has resigned to enter actuarial work.

MR. C. GOUWNES, of the University of Kansas, has been appointed assistant professor of mathematics at the State College of Iowa.

DR. ANNA M. HOWE has been appointed instructor in mathematics at the Sophie Newcomb College of Tulane University.

MR. H. B. MEEK has been appointed instructor in mathematics at Yale University.

MISS MARIAN M. TORREY has been appointed instructor in mathematics at the University of West Virginia.

MISS MAY J. SPERRY, of Brown University, has been appointed instructor in mathematics and physics at Knox College, Galesburg, Ill.

THE death is reported of the Indian mathematician, SRINIVASA RAMANUJAN, F. R. S., fellow of Trinity College, Cambridge, at the age of thirty-two years.

PROFESSOR P. VAN GEER died at the Hague, on October 3, 1919, at the age of seventy-eight years.

PROFESSOR M. HAID, of the Karlsruhe technical school, died November 15, 1919.

THE death is reported of Professor A. BOERSCH, at Homburg.

THE death is reported of Professor R. HEGER, at Dresden, at the age of seventy-three years.

PROFESSOR PAUL STÄCKEL, of the University of Heidelberg, died December 13, 1919, at the age of fifty-seven years.

THE death is reported of Professor R. MALSTROEM, of the department of mechanics at the technical school at Helsingfors.

BOOK CATALOGUES: Buchhandlung Gustav Fock, Leipzig, Handapparate und Sammlungen von Dissertationen und Programmen auf dem Gebiete der Naturwissenschaften sowie der Mathematik.—Gauthier-Villars, Paris, Bulletin des Publications nouvelles, 4e trimestre, 1919.—Rudolf Geering, Basel, Antiquariats-Katalog Nr. 383 (containing the library of the late Professor J. H. Graf, of Bern).—B. G. Teubner, Leipzig, Neuerscheinungen und Neuauflagen, 1914–1919.

NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

CHILD (J. M.). The early mathematical manuscripts of Leibniz. Translated from the Latin texts published by Carl Immanuel Gerhardt with critical and historical notes by J. M. Child. Chicago and London, Open Court, 1920. 8vo. 4 + 238 pp.

EUCLID. See HEATH (T. L.).

GERHARDT (C. I.). See CHILD (J. M.).

GROSSE (W.). Graphische Papiere und ihre vielseitige Anwendung. Düren, Schleicher und Schüll, 1919. 179 pp.

HEATH (T. L.). Euclid in Greek. Book I, with introduction and notes. Cambridge, University Press, 1920. 12mo. 240 pp. 10s.

LABOUREUR (M.). Cours d'exercices sur le calcul mathématique, algébrique, différentiel et intégral. Paris et Liège, Béranger, 1920. 8vo. 8 + 318 pp. Fr. 30.00

LEIBNIZ (G. W.). See CHILD (J. M.).

II. ELEMENTARY MATHEMATICS.

AMMERMAN (C.). See FORD (W. B.).

BALL (K. F.) and WEST (M. E.). Household arithmetic. (Lippincott's unit texts.) Philadelphia, Lippincott, 1920. 8vo. 271 pp.

BEHRENDSEN (O.) und GÖTTING (E.). Lehrbuch der Mathematik nach modernen Grundsätzen. Oberstufe. Ausgabe A. 2te Auflage. Leipzig, Teubner, 1919. 265 pp. M. 5.00

BETTINI (B.) e CIAMBERLINI (C.). Elementi di artimetica e geometria, per la 2a classe ginnasiale. 3a edizione riveduta e migliorata. Firenze, R. Bemporad e figlio (M. Ricci), 1919. 16mo. 156 pp. L. 3.00

BOWYER (L. H.) and MCINTOSH (C.). Eighth year arithmetic adapted to the Illinois state course of study. Sixth general revision. Monticello, Ill., McIntosh, 1919. 8vo. 151 pp. $0.60

CIAMBERLINI (C.). See BETTINI (B.).

CLARK (J. R.). See RUGG (H. O.).

FORD (W. B.) and AMMERMAN (C.). Second course in algebra. New York, Macmillan, 1920. 9 + 299 pp. $1.28

GAZZANIGA (P.). See VERONESE (G.).

GÖTTING (E.). See BEHRENDSEN (O.).

HART (C.) and WATTS (M. J.). Commercial and industrial arithmetic for students of high school grade. New York, Appleton, 1919. 8vo. 6 + 438 pp. $2.00

MCINTOSH (C.). See BOWYER (L. H.).

RUGG (H. O.) and CLARK (J. R.). Fundamentals of high school mathematics; a text-book designed to follow arithmetic. Yonkers-on-Hudson, World Book Company, 1919. 12mo. 15 + 368 pp. $1.60

SCHWERING (K.). Sammlung von Aufgaben aus der Arithmetik für höhere Lehranstalten. 1ter Lehrgang. 4te Auflage. Freiburg, Herder, 1918. 63 pp. Geb. M. 2.00

SPERA (S.). Elementi di algebra, per le scuole techniche e medie inferiori. 2a edizione, riveduta e corretta. Torino, ditta G. B. Paravia e C. (Milano, stamp. ed. Lombarda, di L. Mondaini), 1919. 16mo. 119 pp. L. 3.00

TESTI (G. M.). Corso di aritmetica con 500 esercizi, ad uso degli alunni delle scuole tecniche, dei ginnasi inferiori e delle scuole complementari. 22a edizione riveduta. Livorno, Giusti, 1920. 16mo. 14 + 307 pp. L. 4.40

——. Corso di matematiche ad uso delle scuole secondarie superiori e più specialmente degli istituti tecnici. Volume 2: Algebra elementare. 9a edizione riveduta. Livorno, Giusti, 1920. 8vo. 11 + 431 pp. L. 7.60

VERONESE (G.). Nozioni elementari di geometria intuitiva ad uso dei ginnasi inferiori. 5a edizione. Padova, fratelli Drucker (Società coop. tipografica), 1919. 8vo. 100 pp.

VERONESE (G.) e GAZZANIGA (P.). Elementi di geometria intuitiva, ad uso delle scuole tecniche e complementari. 4a edizione. Padova, fratelli Drucker (Società coop. tipografica), 1919. 8vo. 8 + 163 pp. L. 3.00

WATTS (M. J.). See HART (C.).

WEBBER (W. B.). Elementary applied mathematics. A practical course for general students. New York, Wiley, 1920. 12mo. 10 + 115 pp. $1.25

WEST (M. E.). See BALL (K. F.).

III. APPLIED MATHEMATICS.

AMALDI (U.). Meccanica razionale: lezioni tenute al corso d'integrazione, aprile-luglio 1919, nella r. scuole d'applicazione per gli ingegneri di Padova. Padova, Parisotto, 1919. 8vo. 411 pp.

ANDREWS (S. T. G.) and BENSON (S. F.). The theory and practice of aeroplane design. London, Chapman and Hall, 1920. 8vo. 454 pp. 15s. 6d.

BARKER, (E. H.). Applied mathematics for high school. Boston, Allyn and Bacon, 1919.

BENSON (S. F.). See ANDREWS (S. T. G.).

BERGET (A.). Topographie. Paris, Larousse, 1920. 8vo. 328 pp. Fr. 12.00

CHINI (M.). Corso speciale di matematiche con numerose applicazioni ad uso dei chemici e dei naturalisti. 4a edizione. Livorno, R. Giusti, 1920. 8vo. 12 + 297 pp. L. 8.50

CONCILIIS (E. DE). Lezioni di meccanica applicata alle costruzioni. Parte 1: Introduzione. (R. Scuola superiore politecnica di Napoli.) Napoli, tip. F. Lubrano, 1919. 8vo. 7 + 112 pp. L. 7.50

GRASSI (G.). Principî scientifici della elettrotecnica; introduzione al corso di elettrotecnica. 4a edizione. (Grande biblioteca tecnica, No. 7.) Torino, soc. tip. ed. Nazionale, 1920. 8vo. 8 + 336 pp. L. 20.00

JUNG (H.). See Nölke (F.).

LEGRAND (L.). Cours de mécanique rationnelle avec de nombreuses applications à l'usage des ingénieurs. Paris et Liège, Béranger, 1920. 8vo. 364 pp. Fr. 48.00

NÖLKE (F.). Das Problem der Entwicklung unseres Planetensystems. Eine kritische Studie. 2te völlig umgearbeitete Auflage. Mit einem Geleitwort von H. Jung. Berlin, Springer, 1919. M. 30.80

OSEEN (C. W.). Atomiska föreställningar i nutidens fysik. Tid, rum och materia. Femton föreläsningar. Stockholm, Albert Bonniers Förlag, 1919.

PEEL (T.). Examples in heat and heat engines. Cambridge, University Press, 1919. 3+104 pp. 5s.

POLSTER (H.). Kinematik. Neudruck. Berlin, 1919. M. 1.25

ROSE (W. N.). Mathematics for engineers. Part 2. London, Chapman and Hall, 1920. 8vo. 419 pp. 13s. 6d.

SECRIST (H.). Statistics in business. New York, McGraw-Hill, 1919. 8vo. 9 + 137 pp.

STEVENS (A. B.). Arithmetic of pharmacy. 4th edition revised and enlarged. New York, Van Nostrand, 1920. 8vo. 110 pp. $1.50

VINAL (E. R.). Mathematics for the accountant. New York, Biddle Publishing Company, 1920. 8vo. 16 + 172 pp. $2.50

TWENTY-NINTH ANNUAL LIST OF PAPERS.

READ BEFORE THE AMERICAN MATHEMATICAL SOCIETY AND SUBSEQUENTLY PUBLISHED, INCLUDING REFERENCES TO THE PLACES OF THEIR PUBLICATION.

ALEXANDER, J. W. Note on two three-dimensional manifolds with the same group. Read Sept. 2, 1919. *Transactions of the American Mathematical Society*, vol. 20, No. 4, pp. 339–342; Oct., 1919.

—— A proof of Jordan's theorem about a simple closed curve. Read April 29, 1916. *Annals of Mathematics*, ser. 2, vol. 21, No. 3, pp. 180–184; March, 1920.

—— Note on Riemann spaces. Read Feb. 28, 1920. *Bulletin of the American Mathematical Society*, vol. 26, No. 8, pp. 370–372; May, 1920.

ALGER, P. L. See VEBLEN, O.

ALTSHILLER-COURT (N.). On a pencil of nodal cubics. Read Dec. 31, 1919. *Bulletin of the American Mathematical Society*, vol. 26, No. 5, pp. 203–211; Feb., 1920.

—— On the orthocentric quadrilateral. Read Dec. 31, 1919. *American Mathematical Monthly*, vol. 27, No. 5, pp. 199–202; May, 1920.

BARNETT, I. A. Integro-differential equations with constant limits of integration. Read Dec. 28, 1918. *Bulletin of the American Mathematical Society*, vol. 26, No. 5, pp. 193–203; Feb., 1920.

BAUER, L. A. Geophysics at the Brussels meetings, July 18–28, 1919. Read Sept. 4, 1919. *Science*, new ser., vol. 50, No. 1296, pp. 399–403; Oct. 31, 1919.

BELL, E. T. The twelve elliptic functions related to sixteen doubly periodic functions of the second kind. Read (San Francisco) April 10, 1920. *Messenger of Mathematics*, new ser., vol. 49, Nos. 5–6, pp. 78–84; Sept. and Oct., 1919.

—— On the number of representations of $2n$ as a sum of $2r$ squares. Read (San Francisco) April 5, 1919. *Bulletin of the American Mathematical Society*, vol. 26, No. 1, pp. 19–25; Oct., 1919.

—— Sur les représentations propres par quelques formes quadratiques de Liouville. Read (San Francisco) Oct. 25, 1919. *Comptes Rendus de l'Académie des Sciences*, vol. 169, No. 17, pp. 711–712; Oct. 27, 1919.

—— On the enumeration of proper and improper representations in homogeneous forms. Read (San Francisco) Oct. 25, 1919. *Annals of Mathematics*, ser. 2, vol. 21, No. 3, pp. 166–179; March, 1920.

BENNETT, A. A. The sign of the distance in analytical geometry. Read Oct. 25, 1919. *American Mathematical Monthly*, vol. 26, No. 8, pp. 344–350; Oct., 1919.

—— Poncelet polygons in higher space. Read Dec. 30, 1919. *Bulletin of the American Mathematical Society*, vol. 26, No. 6, pp. 274–275; March, 1920.

BIRKHOFF, G. D. Recent advances in dynamics. Read Dec. 30, 1919. *Science*, new ser., vol. 51, No. 1307, pp. 51–55; Jan. 16, 1920.

BLISS, G. A. The use of adjoint systems in the problem of differential corrections for trajectories. Read March 29, 1919. *Journal of the United States Artillery*, vol. 51, No. 3, pp. 296–311; Sept., 1919.

—— A method of computing differential corrections for a trajectory. Read March 29, 1919. *Journal of the United States Artillery*, vol. 51, No. 4, pp. 445–449; Oct., 1919.

—— Differential equations containing arbitrary functions. Read Sept. 3, 1919. *Transactions of the American Mathematical Society*, vol. 21, No. 2, pp. 79–92; April, 1920.

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